DAY EIGHTEEN

Unit Test 3

(Waves and Oscillations)

1 When an observer moves towards a stationary source with a certain velocity, he observes an apparent frequency f. When he moves away from the source with same velocity, the observed frequency is $0.8 f$. If the velocity of sound is v, then the speed of the observer is

(a) $\frac{v}{4}$ (b) $\frac{v}{8}$

(c) $\frac{v}{9}$ (d) None of these

2 A transverse wave $y = 0.05 \sin(20\pi x - 50\pi t)$ in metre, is propagating along positive X-axis on a string. A light insect starts crawling on the string with the velocity of 5 cms^{-1} at $t = 0$ along the positive X-axis from a point where $x = 5$ cm. After 5s the difference in the phase of its position is equal to

(a) 150π (b) 250π

(c) 10π (d) 5π

3 Equation of a progressive wave is given by

$$y = 0.2 \cos \pi \left(0.04\, t + 0.02 x - \frac{\pi}{6}\right)$$

The distance is expressed in cm and time in second. What will be the minimum distance between two particles having the phase difference of $\frac{\pi}{2}$?

(a) 4 cm (b) 8 cm

(c) 25 cm (d) 12.5 cm

4 Two waves represented by the following equations are travelling in the same medium

$$y_1 = 5 \sin 2\pi (75 t - 0.25 x)$$
$$y_2 = 10 \sin 2\pi (150 t - 0.50 x)$$

The intensity ratio $\frac{I_1}{I_2}$ of the two waves is

(a) 1 : 2 (b) 1 : 4

(c) 1 : 8 (d) 1 : 16

5 Two sources of sound A and B produces the wave of 350 Hz, in the same phase. The particle P is vibrating under the influence of these two waves, if the amplitudes at the point P produces by the two waves is 0.3 mm and 0.4 mm, then the resultant amplitude of the point P will be when $AP - BP = 25$ cm and the velocity of sound is 350 ms^{-1}.

(a) 0.7 mm (b) 0.1 mm (c) 0.2 mm (d) 0.5 mm

6 Equation of a plane progressive wave is given by $y = 0.6 \sin 2\pi \left(t - \frac{x}{2}\right)$. On reflection from a denser medium, its amplitude becomes 2/3 of the amplitude of the incident wave. The equation of the reflected wave is

(a) $y = 0.6 \sin 2\pi \left(t + \frac{x}{2}\right)$ (b) $y = -0.4 \sin 2\pi \left(t + \frac{x}{2}\right)$

(c) $y = 0.4 \sin 2\pi \left(t + \frac{x}{2}\right)$ (d) $y = -0.4 \sin 2\pi \left(t - \frac{x}{2}\right)$

7 Two travelling waves $y_1 = A \sin[k(x - ct)]$ and $y_2 = A \sin[k(x + ct)]$ are superimposed on string. The distance between adjacent nodes is

(a) $\frac{c}{\pi}$ (b) $\frac{c}{2\pi}$ (c) $\frac{\pi}{2k}$ (d) $\frac{\pi}{k}$

8 A source of sound is moving with constant velocity of 20 ms^{-1} emitting a note of frequency 1000 Hz. The ratio of frequencies observed by a stationary observer while the source is approaching him and after it crosses him will be (speed of sound, $v = 340$ ms^{-1})

(a) 9 : 8 (b) 8 : 9 (c) 1 : 1 (d) 9 : 10

9 A table is revolving on its axis at 5 rev/s. A sound source of frequency 1000 Hz is fixed on the table at 70 cm from the axis. The minimum frequency heard by a listener standing at a distance very far from the table will be (speed of sound, $v = 352$ ms^{-1})

(a) 1000 Hz (b) 1066 Hz

(c) 941 Hz (d) 352 Hz

10 The wavelength is 120 cm when the source is stationary. If the source is moving with relative velocity of 60 ms^{-1} towards the observer, then the wavelength of the sound wave reaching to the observer will be
(velocity of sound, $v = 330\ ms^{-1}$)

(a) 98 cm (b) 140 cm (c) 120 cm (d) 1440 cm

11 An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

(a) 5% (b) 20% (c) zero d) 0.5%

12 In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

(a) 0.012 m (b) 0.025 m (c) 0.05 m (d) 0.024 m

13 Two bodies M and N of equal masses are suspended from two separate springs of constants k_1 and k_2, respectively. If they oscillate with equal maximum velocities, then the amplitudes of M and N are in the ratio of

(a) k_1/k_2 (b) k_2/k_1 (c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$

14 Two simple pendulums of length 0.5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed, how many oscillations?

(a) 5 (b) 1 (c) 2 (d) 3

15 A particle is executing SHM with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cms^{-1}. The distance of the particle from the mean position when its speed becomes 5 cms^{-1} is

(a) $\sqrt{3}$ cm (b) $\sqrt{5}$ cm (c) $2\sqrt{3}$ cm (d) $2\sqrt{5}$ cm

16 In a Kundt's tube experiment, the distance between nodes was 8 cm. If velocity of sound in air is 330 ms^{-1} and total length of sounding rod is 160 cm, then what is the velocity of sound in rod?

(a) 6800 ms^{-1} (b) 7200 ms^{-1}
(c) 6200 ms^{-1} (d) 6600 ms^{-1}

17 A particle executes SHM between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then,

(a) $T_1 < T_2$ (b) $T_1 > T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

18 A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

19 The displacement of a particle varies with time according to the relation $y = a\sin\omega t + b\cos\omega t$.

(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

20 The length of a simple pendulum executing SHM is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 10% (b) 11% (c) 21% (d) 42%

21 Two tuning forks when sounded together produce 10 beats/s. The first fork has the frequency 5% less than the standard fork and the second has the frequency 8% higher than a standard one. What are the frequencies of the forks?

(a) 73 Hz, 83 Hz (b) 83 Hz, 93 Hz
(c) 103 Hz, 93 Hz (d) 113 Hz, 103 Hz

22 A source of sound gives five beats per second, when sounded with another source of frequency $100\ s^{-1}$. The second harmonic of the source, together with a source of frequency $205\ s^{-1}$ gives five beats per second. What is the frequency of the source?

(a) $105\ s^{-1}$ (b) $205\ s^{-1}$ (c) $95\ s^{-1}$ (d) $100\ s^{-1}$

23 A column of air at 88°C and a tuning fork produce 6 beats/s when sounded together. When temperature is 16°C the two produce only one beat per second. What is the frequency of tuning fork?

(a) 53.9 Hz (b) 97.7 Hz (c) 41.5 Hz (d) 87.7 Hz

24 A steel wire of length 80 cm and area of cross-section $10^{-5}\ m^2$ is joined with an aluminium wire of length 60 cm and area of cross-section $3 \times 10^{-5}\ m^2$. The composite string is stretched by a tension 100 N. What is minimum frequency of tuning fork, which can produce standing wave in it with node at the joint?
(Given, $\rho_{steel} = 7800\ kg\ m^{-3}$ and $\rho_{al} = 2600\ kg\ m^{-3}$)

(a) 87.5 Hz (b) 97.8 Hz (c) 67.9 Hz (d) 89.5 Hz

25 A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in

(a) 1 s (b) 0.5 s
(c) 2 s (d) Data insufficient

26 A spring of force constant k is cut into two pieces such that one piece is doubled the length of the other. Then, the longer piece will have a force constant

(a) $2k/3$ (b) $3k/2$ (c) $3k$ (d) $6k$

27 The potential energy of a simple harmonic oscillator when the particle is half way to its end point is

(a) $\frac{1}{8}E$ (b) $\frac{1}{4}E$ (c) $\frac{1}{2}E$ (d) $\frac{2}{3}E$

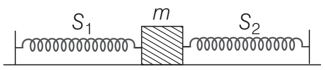
28 An elastic ball is dropped from a height h on an elastic floor. It collides against the floor elastically and rebounds to the same height. What is the nature of its motion and time period of return to the starting point?

(a) SHM, $\sqrt{8h/g}$ (b) Oscillatory, $\sqrt{8h/g}$
(c) SHM, $2\sqrt{h/g}$ (d) Oscillatory, $2\sqrt{h/g}$

29 In a transverse progressive wave of amplitude A, the maximum particle velocity is four times the wave velocity. Then, the wavelength of the wave is

(a) $\frac{\pi A}{2}$ (b) $\frac{\pi A}{4}$ (c) πA (d) $2\pi A$

30 When a mass m is connected individually to two springs S_1 and S_2, the oscillation frequencies are ν_1 and ν_2. If the same mass is attached to the two springs as shown in figure the oscillation frequency would be



(a) $\nu_1 + \nu_2$ (b) $\sqrt{\nu_1^2 + \nu_2^2}$ (c) $\left(\frac{1}{\nu_1} + \frac{1}{\nu_2}\right)^{-1}$ (d) $\sqrt{\nu_1^2 - \nu_2^2}$

31 The equation of a simple harmonic is given by

$$y = 3 \sin \frac{\pi}{2}(50\,t - x),$$

where, x and y are in metres and t is in seconds. The ratio of maximum particle velocity to the wave velocity is

(a) 2π (b) $\frac{3}{2}\pi$ (c) 3π (d) $\frac{2}{3}\pi$

32 A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity 4ms^{-1}. It collides with a horizontal spring of force constant 200 Nm^{-1}. The maximum compression produced in the spring will be

(a) 0.5 m (b) 0.6 m (c) 0.7 m (d) 0.2 m

33 The amplitude of a particle executing SHM is 4 cm. As the mean position, the speed of the particle is 16 cm/s. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cm/s will be

(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm (c) 1 cm (d) 2 cm

34 A simple harmonic oscillator consists of a particle of mass m and an ideal spring with spring constant k. The particle oscillates with a time period T. The spring is cut into two equal parts. If one part oscillates with the same particle, the time period will be

(a) $2T$ (b) $\sqrt{2T}$ (c) $\frac{T}{\sqrt{2}}$ (d) $\frac{T}{2}$

35 A train moving at a speed of 220 ms^{-1} towards a stationary object, emits a sound of frequency 1000 Hz. Some of the sound reaching the object gets reflected back to the train as echo. The frequency of the echo as detected by the driver of the train is (speed of sound in air is 330 ms^{-1})

(a) 3500 Hz (b) 4000 Hz (c) 5000 Hz (d) 3000 Hz

36 When a certain volume of water is subjected to increase of 100 kPa pressure, the volume of water decreases by 0.005%. The speed of sound in water must be

(a) 140 m/s (b) 300 m/s (c) 1400 m/s (d) 5000 m/s

37 A progressive wave moving along X-axis is represented by $y = A \sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$. The wavelength ($\lambda$) at which the maximum particle velocity is 3 times the wave velocity is

(a) $\frac{A}{3}$ (b) $\frac{2A}{(3\pi)}$ (c) $\left(\frac{3}{4}\right)\pi A$ (d) $\left(\frac{2}{3}\right)\pi A$

38 The frequency of the first overtone of a closed pipe of length l_1 is equal to that of the first overtone of an open pipe of length l_2. The ratio of their lengths ($l_1 : l_2$) is

(a) 2 : 3 (b) 4 : 5
(c) 3 : 5 (d) 3 : 4

39 A particle is executing SHM. Then, the graph of velocity as a function of displacement is a/an

(a) straight line (b) circle
(c) ellipse (d) hyperbola

40 A long spring, when stretched by a distance x, has potential energy U. On increasing the stretching to nx, the potential energy of the spring will be

(a) $\frac{U}{n}$ (b) nU
(c) $n^2 U$ (d) $\frac{U}{n^2}$

41 If the maximum velocity and acceleration of a particle executing SHM are equal in magnitude the time period will be

(a) 1.57 s (b) 3.14 s
(c) 6.28 s (d) 12.56 s

42 Two pendulums have time period T and $5T/4$. They start SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completed one oscillation?

(a) 45° (b) 90°
(c) 60° (d) 30°

43 A sound has an intensity of $2 \times 10^{-8}\text{ W/m}^2$. Its intensity level (in decibel) is ($\log_{10} 2 = 0.3$)

(a) 23 (b) 3
(c) 43 (d) 4.3

44 Two waves having the intensities in the ratio of 9:1 produce interference. The ratio of maximum to minimum intensity is equal to

(a) 10 : 8 (b) 9 : 1
(c) 4 : 1 (d) 2 : 1

45 Which of the following is independent from others?

(a) Wavelength (b) Velocity
(c) Frequency (d) Amplitude

Direction (Q. Nos. 46-50) *In each of the following questions a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements mark the correct answer as*

(a) If both Assertion and Reason are true and Reason is the correct explanation of assertion
(b) If both Assertion and Reason are true but the Reason is not the correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If both Assertion and Reason are false

46 **Assertion** (A) Simple Harmonic Motion (SHM) is not a uniform motion.

Reason (R) It is the projection of uniform circular motion.

47 **Assertion** (A) A hole were drilled through the centre of earth and a ball is dropped into the hole at one end, it will not get out of other end of the hole.
Reason (R) It will come out of the other end normally.

48 **Assertion** (A) A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.
Reason (R) The same tuning fork will not be in resonance with open pipe of same length due to end correction of pipe.

49 **Assertion** (A) In everyday life, the Doppler's effect is observed readily for sound waves than light waves.

Reason (R) Velocity of light is greater than the sound.

50 **Assertion** (A) Velocity of particles while crossing mean position (in stationary waves) varies from maximum at anti-nodes to zero at nodes.

Reason (R) Amplitude of vibration at anti-nodes is maximum and at nodes, the amplitude is zero and all particles between two successive nodes cross the mean position together.

ANSWERS

1. (c)	**2.** (d)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (a)
21. (a)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (b)
31. (b)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (d)	**39.** (c)	**40.** (c)
41. (c)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (b)	**50.** (a)

Hints and Explanations

1 Suppose, f_0 be the actual frequency.
Then,

$$f = f_0\left(\frac{v + v_o}{v}\right) \quad \ldots\text{(i)}$$

and $$0.8f = f_0\left(\frac{v - v_o}{v}\right) \quad \ldots\text{(ii)}$$

On solving these two equations, we get,
$v_o = \frac{v}{9}$

2 $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = k(\Delta x) = k(vt)$

Here, vt = distance travelled by insect in given time interval.

or $\Delta\phi = (20\pi)(5\times10^{-2}\times5) = 5\pi$

3 On comparing with
$y = a\cos(\omega t + kx - \phi)$, we get

$$k = \frac{2\pi}{\lambda} = 0.02\pi$$

$\Rightarrow \quad \lambda = 100$ cm

$$\Delta\phi = \frac{\pi}{2}$$

Hence, path difference between them,

$$\Delta x = \frac{\lambda}{2\pi}\times\Delta\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{2}$$
$$= \frac{\lambda}{4} = \frac{100}{4} = 25\text{ cm}$$

4 $\therefore \frac{I_1}{I_2} = \frac{A_1^2\omega_1^2}{A_2^2\omega_2^2} = \left(\frac{5}{10}\right)^2\left(\frac{2\pi\times75}{2\pi\times150}\right)^2 = \frac{1}{16}$

5 $\therefore \lambda = \frac{v}{f} = \frac{350}{350} = 1\text{ m} = 100$ cm

Also, path difference (Δx) between the waves at the point of observation is $AP - BP = 25$ cm. Hence,

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{1}\times\left(\frac{25}{100}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad A = \sqrt{(a_1)^2 + (a_2)^2}$$
$$= \sqrt{(0.3)^2 + (0.4)^2} = 0.5\text{ mm}$$

6 By reflection from a denser medium phase reversal $= 180°$
New amplitude $= \frac{2}{3}\times0.6 = 0.4$
Hence, new equation for the reflected wave
$$y = 0.4\sin2\pi\left[t + \frac{x}{2} + 180°\right]$$
$$= -0.4\sin2\pi(t + x/2)$$

7 The distance between adjacent nodes
$$x = \frac{\lambda}{2}$$
Also, $k = \frac{2\pi}{\lambda}$

Hence, $x = \frac{\pi}{k}$

8 $\frac{f_a}{f_r} = \frac{v + v_s}{v - v_s} = \frac{340 + 20}{340 - 20} = \frac{9}{8}$

9 For source v_s
$= r\omega = 0.70\times2\pi\times5 = 22\text{ ms}^{-1}$

Minimum frequency is heard when the source is receding the man. It is given by

$$f_{\min} = f\, \frac{v}{v + v_s}$$

$$= 1000 \times \frac{352}{352 + 22} = 941 \text{ Hz}$$

10 $$f' = f\left(\frac{v}{v - v_s}\right)$$

$$\Rightarrow \quad \lambda' = \lambda\left(\frac{v - v_s}{v}\right)$$

$$\Rightarrow \quad \lambda' = 120\left(\frac{330 - 60}{330}\right) = 98 \text{ cm}$$

11 When observer moves towards stationary source, then apparent frequency

$$f' = \left[\frac{v + v_o}{v}\right] f = \left[\frac{v + v/5}{v}\right] f$$

$$= \frac{6}{5} f = 1.2\, f$$

Increment in frequency $= 0.2\, f$.

So, percentage change in frequency

$$= \frac{0.2f}{f} \times 100 = 20\%$$

12 Let x be the end correction, then according to question.

$$\frac{v}{4\,(l_1 + x)} = \frac{3v}{4\,(l_2 + x)}$$

$$\Rightarrow \quad x = 2.5 \text{ cm} = 0.025 \text{ m}$$

13 $v_{\max} = A\omega$, for mass attached to spring $\omega = \sqrt{k/M}$. Here, masses are equal.

Therefore, $A_1\,\omega_1 = A_2\omega_2$

or $A_1\sqrt{k_1} = A_2\sqrt{k_2}$

i.e. $$\frac{A_1}{A_2} = \left[\frac{k_2}{k_1}\right]^{1/2} = \sqrt{\frac{k_2}{k_1}}$$

14 $nT_s = (n - 1)\,T_1$

$$n\, 2\pi\sqrt{\frac{0.5}{g}} = (n - 1)\, 2\pi\sqrt{\frac{20}{g}}$$

or $n = (n - 1)\sqrt{40} \cong 6\,(n - 1)$

Hence, $n = 6/5 \cong 1$

15 Here, $v = \omega\,[A^2 - y^2]^{1/2}$

or $v^2 = \omega^2\,[A^2 - y^2]$

Also $v_{\max} = \omega A$

i.e. $10 = \omega \times 4$

$\Rightarrow \quad \omega = 2.5$

Putting $v = 5 \text{ cms}^{-1}, \omega = 2.5$

and $A = 4$ cm

$$(5)^2 = (2.5)^2[(4)^2 - (y)^2]$$

$$\Rightarrow \quad \frac{25}{2.5 \times 2.5} = (4)^2 - (y)^2$$

$\Rightarrow y^2 = 16 - 4$

$\Rightarrow \; y^2 = 12 \Rightarrow y = 2\sqrt{3}$ cm

16 $$\frac{\text{Velocity of sound in solid}}{\text{Velocity of sound in air}} = \frac{\text{Total length of rod}}{\text{Distance between nodes}}$$

$$\frac{v_{\text{solid}}}{330} = \frac{160 \times 10^{-2}}{8 \times 10^{-2}}$$

Therefore, $v_{\text{solid}} = 6600 \text{ ms}^{-1}$

17 Let the equation of SHM be

$y = A \sin \omega t$

Then, in the first case $\frac{A}{2} = A \sin\omega\, T_1$

i.e. $\omega T_1 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

Therefore, $T_1 = \frac{\pi}{6\omega}$

Also, $T_2 = \frac{T}{4} - T_1 = \frac{2\pi}{4\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$

Therefore, $T_1 < T_2$

18 Time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{GM/r^2}}$$

$$= 2\pi r\sqrt{\frac{l}{GM}}$$

Hence, $T_1 = 2\pi R\sqrt{\frac{l}{GM}}$

And $T_2 = 2\pi\,(2R)\sqrt{\frac{l}{GM}}$

Therefore, $\frac{T_2}{T_1} = 2$

19 $y = a\sin\omega t + b\cos\omega t$

Let $a = A\cos\theta\,;\, b = A\sin\theta$

$\Rightarrow \quad a^2 + b^2 = A^2\,(\cos^2\theta + \sin^2\theta) = A^2$

or $A = \sqrt{a^2 + b^2}$

Hence, $Y = A\cos\theta\sin\omega t + A\sin\theta\cos\omega t$

$= A\sin(\omega t + \theta)$

$= \sqrt{a^2 + b^2}\, \sin(\omega t + \theta)$

It is SHM with amplitude $A = \sqrt{a^2 + b^2}$

20 Time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots\text{(i)}$$

21% increase $l \Rightarrow l' = l + \frac{21}{100}\, l = \frac{121}{100}\, l$

Therefore, $T' = 2\pi\sqrt{\frac{l'}{g}} = 2\pi\sqrt{\frac{121/100l}{g}}$

$$= \frac{11}{10} T \quad \text{[using Eq. (i)]}$$

Hence, percentage increase in the time period,

$$\frac{T' - T}{T} \times 100 = \frac{\frac{11}{10} T - T}{T} \times 100 = 10\%$$

21 Let, frequency of standard fork $= \nu$

Frequency of first fork, $\nu_1 = \frac{95}{100}\nu$

Frequency of second fork, $\nu_2 = \frac{108}{100}\nu$

Number of beats $= \nu_2 - \nu_1$

$\Rightarrow \quad \nu_2 - \nu_1 = 10$

or $\frac{108}{100}\nu - \frac{95}{100}\nu = 10$

or $\frac{13}{100}\nu = 10$

or $\nu = 76.92 \cong 77$ Hz

Therefore, $\nu_1 = \frac{95}{100} \times 77 = 73$ Hz

$\nu_2 = \frac{108}{100} \times 77 = 83$ Hz

22 The frequency of source can be $100 + 5$ and $100 - 5$

i.e. 105 or 95

2nd harmonic is 210 or 200.

Since, 5 beats with 105 are possible, if source frequency is 210.

$\therefore$ Original frequency is 105 s^{-1}.

23 We know that $\nu \propto v$ and $v \propto \sqrt{T}$

Therefore, $\frac{\nu_{88}}{\nu_{16}} = \frac{v_{88}}{v_{16}} = \sqrt{\frac{273 + 88}{273 + 16}} = \frac{19}{17}$

As $\nu_{16} < \nu_{88}$, hence frequency of tuning fork ν is less than the frequency of air column.

Therefore, $\nu_{88} = \nu + 6$

and $\nu_{16} = \nu + 1$

$\therefore \quad \frac{\nu + 6}{\nu + 1} = \frac{19}{17}$

On solving, we get $\nu = 41.5$ Hz

24 The frequency of the stretched string is given by

$$\nu = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

Therefore,

$$\nu_{\text{steel}} = \frac{n_1}{2 \times 80 \times 10^{-2}}\sqrt{\frac{100}{7800 \times 10^{-5}}} \quad \ldots\text{(i)}$$

and

$$\nu_{\text{al}} = \frac{n_2}{2 \times 60 \times 10^{-2}}\sqrt{\frac{100}{3 \times 10^{-5} \times 2600}} \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$\frac{n_1}{n_2} = \frac{4}{3}$$

$$\nu_{\text{steel}} = \frac{4}{2 \times 0.8}\sqrt{\frac{100}{7800 \times 10^{-5}}} = 89.5 \text{ Hz}$$

[from Eq. (i)]

In composite string, different harmonics give rise to a common frequency.

$\therefore \quad \nu_{\text{steel}} = \nu_{\text{al}} = 89.5$ Hz

25 m = mass per unit length $=\frac{2.5}{20}$ kg/m

$T = 200$ N

$$v=\sqrt{\frac{T}{m}}=\sqrt{\frac{200}{2.5/20}}=\sqrt{\frac{200\times 200}{25}}$$

$$=\frac{200}{5}=40 \text{ m/s}$$

$\therefore$ Required time $=\frac{i}{v}=\frac{20}{40}=0.5$ s

26 As the spring is cut into two parts of length $\frac{l}{3}, \frac{2l}{3}$

We know that, $k=\frac{mg}{x}$, here x is extension

For spring of length $\frac{l}{3}$ extension $= x\frac{l}{3}$

For spring of length $\frac{2l}{3}$ extension $= x\frac{2l}{3}$

For spring of length $\frac{2l}{3}$, we have spring constant $\frac{3}{2}k$

27 Potential energy, $E = 1/2m\,\omega^2 y^2$

At end point, $y = A$

Therefore, $E = 1/2\, m\,\omega^2 A^2$ ...(i)

At half way $y' = A/2$

Therefore, $E' = 1/2m\,\omega^2 (y')^2$

$$= 1/2\, m\,\omega^2 \frac{A^2}{4} \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$E'=\frac{E}{4}$$

28 The motion is under constant acceleration and therefore y is not proportional to displacement. Hence, the motion is oscillatory but not SHM.

Time of fall = Time of rise $= t$ (say)

The time of rise is given by

$$-h = 0\times t - \frac{1}{2}gt^2$$

$$\left[\text{using } s = ut + \frac{1}{2}at^2\right]$$

Hence, $t=\sqrt{\frac{2h}{g}}$

Therefore, time period of oscillation

$$=2\sqrt{\frac{2h}{g}}=\sqrt{\frac{8h}{g}}$$

29 $\omega A = 4\times v$(wave velocity)

or $\omega A = 4v = 4n\lambda$

or $2\pi nA = 4n\lambda$

or $\lambda=\frac{\pi A}{2}$

30 Let k_1 and k_2 be the spring constants of S_1 and S_2, respectively.

Then, $\nu_1=\frac{1}{2\pi}\sqrt{\frac{k_1}{m}}$ and $\nu_2=\frac{1}{2\pi}\sqrt{\frac{k_2}{m}}$

$$k_{eq}=k_1+k_2$$

$$\nu=\frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}}=\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

$$=\frac{1}{2\pi}\sqrt{\frac{k_1}{m}+\frac{k_2}{m}}=\frac{1}{2\pi}\sqrt{4\pi^2\nu_1^2+4\pi^2\nu_2^2}$$

$$\Rightarrow \quad \nu=\sqrt{\nu_1^2+\nu_2^2}$$

31

$$v_{max}=a\omega \text{ and } v=n\lambda$$

$$\frac{v_{max}}{v}=\frac{a\omega}{n\lambda}=\frac{a(2\pi n)}{n\lambda}=\frac{2\pi a}{\lambda}$$

$$=\frac{2\pi a}{2\pi/k}=ka=\frac{\pi}{2}\times 3=\frac{3\pi}{2}$$

32 Loss in KE = Gain in spring energy

$$\frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]=\frac{1}{2}k\,x_{max}^2$$

where, k is the force constant.

Given, $v = 4$ m/s, $m = 3$ kg and $k = 200$ N/m

For solid cylinder, $\frac{k^2}{R^2}=\frac{1}{2}$

$$\therefore \frac{1}{2}\times 3\times (4)^2\left[1+\frac{1}{2}\right]=\frac{1}{2}\times 200\times x_{max}^2$$

The maximum compression in the spring

$$x_{max} = 0.6 \text{ m}$$

33 At mean position, velocity is maximum

$$v_{max}=a\omega$$

$$\omega=\frac{v_{max}}{a}=\frac{16}{4}=4$$

$$\therefore \quad v=\omega\sqrt{a^2-y^2}\Rightarrow 8\sqrt{3}=4\sqrt{4^2-y^2}$$

$$192=16(16-y^2)\Rightarrow 12=16-y^2$$

$$y=2 \text{ cm}$$

34 Mass of the particle $= m$

Spring constant $= k$

The time period of oscillator,

$$T=2\pi\sqrt{\frac{m}{k}}$$

As $k\propto\frac{1}{l}$ [where, l is the length of spring]

$\because \quad k'=2k$

$$\therefore \quad T'=2\pi\sqrt{\frac{m}{2k}}=\frac{1}{\sqrt{2}}T$$

35 From Doppler's shifts, we knows for this case

$$n'=n\left(\frac{v+v_s}{v-v_s}\right)$$

$$=1000\left(\frac{330+220}{330-220}\right)=1000\left(\frac{550}{110}\right)$$

$$=5000 \text{ Hz}$$

36 Volume elasticity

$$B=\frac{\Delta p}{\Delta V/V}=\frac{100\times 10^3}{0.005/100}=2\times 10^9$$

Speed of sound, $v=\sqrt{\frac{B}{d}}$

$$=\sqrt{\frac{2\times 10^9}{10^3}}$$

$$=1400 \text{ m/s}$$

37 Progressive wave, $y=A\sin\left[\frac{2\pi}{\lambda}(vt-x)\right]$

$$v=\frac{2\pi}{\lambda}vA$$

According to the question,

$$v_{max}=3v$$

$$\therefore \quad 3v=\frac{2\pi}{\lambda}vA$$

$$\Rightarrow \quad \lambda=\frac{2\pi}{3}A$$

38 Here, $v_1=\frac{3}{4l_1}v$

$$v_2=\frac{2}{2l_2}v$$

Given, $v_1=v_2$

The ratio of lengths $\frac{l_1}{l_2}=\frac{3}{4}$

39 From the relation, $v^2=\omega^2(a^2-x^2)$

$$\Rightarrow \quad v^2+\omega^2x^2=\omega^2a^2$$

$$\Rightarrow \quad \frac{v^2}{a^2\omega^2}+\frac{x^2}{a^2}=1$$

Which is the equation of an ellipse.

40 Potential energy of the spring,

$$U=\frac{1}{2}kx^2$$

and $U'=\frac{1}{2}k(nx)^2$

$$U'=n^2\frac{1}{2}kx^2=n^2U$$

41 According to question,

$|v_{max}|=|a_{max}|$ i.e. $\omega^2 a=a$,

$$\Rightarrow \quad \omega=1=\frac{2\pi}{T}$$

$$\therefore \quad T=2\pi$$

$$\Rightarrow \quad T=2\times 3.14$$

$$\Rightarrow \quad T=6.28 \text{ s}$$

42 When bigger pendulum of time period $(5T/4)$ completes one vibration, the smaller pendulum will complete (5/4) vibrations. It means the smaller pendulum will be leading, the bigger pendulum by phase $T/4$ second $=\pi/2$ rad = 90°.

43 $L = 10\log_{10}\left(\frac{I}{I_0}\right)$

$L = 10\log_{10}\left(\frac{2\times10^{-8}}{10^{-12}}\right)$

$[\because I_0 = 10^{-12}\ \text{W/m}^2]$

$L = 10\log_{10}(2\times10^4)$

$L = 10[\log_{10}2 + \log_{10}(10)^4]$

$L = 10(4.3) = 43$ dB

44 $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{1}$

$\Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$

Then, $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(3+1)^2}{(3-1)^2}$

$= \frac{(4)^2}{(2)^2} = \frac{16}{4} = \frac{4}{1}$

Thus, $I_{max} : I_{min} = 4 : 1$

45 Amplitude is independent of wavelength, velocity and frequency of oscillation.

46 In Simple Harmonic Motion (SHM),

$$v = \omega\sqrt{(a^2 - y^2)}$$

As y changes, velocity v will also change. So, Simple Harmonic Motion (SHM) is not a uniform motion.

Simple Harmonic Motion (SHM) may be defined as the projection of uniform circular motion along one of the diameters of the circle.

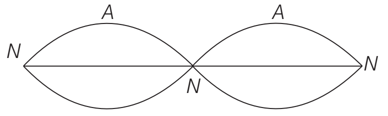
47 The ball will not go out of the other end of the hole, because it will execute SHM. On reaching the other end of the hole, its velocity becomes zero and acceleration of ball will be maximum and will be directed towards the centre of earth.

48 If a closed pipe of length L is in resonance with a tuning fork of frequency ν, then $\nu = \frac{v}{4L}$

An open pipe of same length L produces vibrations of frequency $\frac{v}{2L}$. Obviously, it cannot be in resonance with the given tuning fork of frequency $\nu\left(=\frac{v}{4L}\right)$.

49 Doppler's effect is observed readily in sound waves due to the larger wavelength. This effect is not followed with light due to shorter wavelength. The velocity of light is $3\times10^8\,\text{ms}^{-1}$ and $332\ \text{ms}^{-1}$ velocity of sound.

50 Stationary wave is represented as shown in figure.



It is quite clear from figure that at nodes the amplitude is zero and velocity of particle is also zero and at anti-nodes the amplitude is maximum.

So that the velocity of particle is also maximum and all particles cross mean position between two successive modes.